Appendix F: Estimation Techniques: Logistic and Censored Logistic Regression

The conventional technique for estimating mean willingness-to-pay (WTP) from referendum data involves the estimation of a logistic regression equation of the following form:

$$\ln[P_i/(1-P_i)] = \_\ C_i + \ \_'X_{i\,,}$$

where,

P_i = probability of an affirmative vote,

$\ln[P_i/(1-P_i)]$ = "log odds" of an affirmative vote,

C_i = cost to household i if the policy is adopted,

X_i = other explanatory variables (income, education, etc.) for household i,

$\_\ ,\_$ = model parameters (to be estimated).

Solving for P_i yields:

$$P_i = [1 + exp\text{-}(\_C_i + \ \_'X_i\ )]^{-1},$$

which is the cumulative density function (c.d.f.) of the logistic p.d.f. From this expression the log-likelihood function can be derived and maximized with respect to _ and _. The estimated mean WTP is obtained by integrating the fitted logistic c.d.f. with respect to C_i.

Censored logistic regression represents an extension of the above approach. Censored logistic regression has the advantage of providing an explicit expression for estimated mean WTP in the following form:

$$E(WTP_i) = \_'X_i\ .$$

The _ parameters can be obtained from the parameters of the conventional logistic regression model through the following transformation (Cameron, 1988):

$$\_ = -\_ / \_ \ .$$

Moreover, the asymptotic standard errors of the estimated _ can be obtained from the variance-covariance matrix of the standard logistic regression model using the following transformation (Patterson and Duffield, 1991):

$$Var(\beta_j) = (g_j^2/\_^4)Var(\_) - 2(g_j/\_^3)Cov(\_,g_j) + (1/\_^2)Var(g_j) \qquad \text{(F5)}$$

This yields estimates that are equivalent to those obtained by maximizing the censored logistic log-likelihood function. (Patterson and Duffield, 1991, demonstrate the mathematical equivalence of these two approaches, and provide an example. Hagen, Vincent and Welle, 1992, utilize both approaches and find that the results are identical.) All of the censored logistic regression estimates presented in this report were obtained by transforming the logistic regression parameters in the manner described above.